Application of the logistical and Gompertz models to the analysis of live weight curves in Cuban Creole kids

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Growth curves in Cuban creole kids were studied adjusted to the logistical and Gompertz models through the application of elasticity calculation and their biological interpretations. The elasticity was calculated by the expression $\eta = \frac{dy}{dx} \cdot \frac{x}{y}$. The range where the adjusted model was elastic (> 1) was considered as growth peak period. From the models the adjusted and potential adult weight, the highest growth rate and the age at which is reached, were estimated. In both models significant values were recorded in the adjustments, with coefficients of determination higher than 98% and the typical error of the estimations, with values between 1371 and 1711 g. The adult adjusted weight was of 36. 522 and 31.354 kg for males and females, respectively, according to the logistical model and in regard to the Gompertz model, corresponded to 40.373 and 34.297 kg for males and females, respectively. The analysis of elasticity of both models were effective to estimate the limits and the duration of the growth peak period, which was extended from 165 to 193 d, and ended between 225 and 260 d of age, when the animals attained live weights around 20 and 26 kg, for females and males, respectively. Daily live weight gains were of 70 g for both sexes at the beginning of the growth peak and at its end of 93 to 104 g daily, for females and males, respectively.

Key words: *growth, elasticity, goats, Criollas*

Growth analysis of live beings and their population is subject of consideration in many applied agricultural research studies. Growth is closely related to the physiological and productive performance of cultures and domestic animals and from its analysis different indicators are derived relative to growth rates and the period of maximum expression of this indicator, known as growth peak.

The growth peak, in studies of pastures as in domestic animals, defines periods of better exploitation of the potential of the species in question. From the definition of this period of greater growth expression, important decisions are taken to improve the productive systems (González and Pereda 2004).

Growth curves are adjusted to mathematical models from which predictions of the growth variable can be obtained. However, other interpretations according to the analysis of the model obtained are not frequently exploited.

The elasticity of a function is a coefficient mainly used in economy that allows analyzing the way in which a change in the independent variable affects the function from the percentile point of view (Casa 2007). It is denoted by the Greek letter ŋ (eta). In this case, is the percentile increase of the growth indicator the age increase in 1 % (Ricardo *et al*. 2005). Elasticity higher than 1 % indicates that the biological material under study grows more than it ages.

It is clear that the growth peak cannot be only established on mathematical basis, but the analysis of this indicator could result an important decision criterion.

The objective of this study was to introduce the term elasticity in the logistical and Gompertz models through the study of the growth curves of Cuban Creole kids and also to deepen in their application for determining the growth peak.

Materials and Methods

Localization of the study. A selection of 24 Creole kids of both sexes was made. Animals were the offspring of registered specimens belonging to three breeders of the community "26 de Julio" and to the Genetic Breeding Enterprise "Manuel Fajardo" in Granma province, Cuba. The rearing systems are in correspondence with the traditional rearing typologies of the Criolla goat reported by La O. *et al* (2010). They are based on self-survival and feeding objectives consisting mainly of pastures and natural shrub-likes.

Weighing. Weighing was carried out before leaving to grazing with a monthly frequency, during a period of fourteen months.

Selected models. A non-linear analysis of regression was carried out to adjust the live weight curve, by sexes, utilizing the logistical and Gompertz models where:

Logistical model:

$y = a/(1+b\exp(-cx))$

Gompertz model:

$y = a \exp(-\exp(b-(cx))$

For both models:

y, live weight of kids in grams

x, age in d.

a, b, c parameters of the models

Parameters were estimated according to the Levenberg-Marquardt method, described by Moré

(1977). The evaluation of the adjustment of the models was realized through the comparison of the following statistical criteria: residual variance, significance of the regression, coefficient of determination (R^2), sum of residual squares and typical error of the estimation ($\sqrt{CMe}$).

Calculation of the elasticity and mathematical criteria. The value of the elasticity (ŋ) was calculated by the expression:

$$\eta = \frac{dy}{dx} \cdot \frac{x}{y}$$

where:

$\frac{dy}{dx}$ is the derivative value of the adjusted model.

$y = f(X_0)$ in the point x0.

The adjusted model was considered elastic in the points where the modular value of η was higher than 1 (Ricardo *et al*. 2005). All the range in which the adjusted model was elastic was considered as growth peak period.

Estimations of zootechnical interest from the adjusted model. Table 1 shows other interpretations from the analysis of the properties of both models.

Data processing was carried out using the STATISTICA statistical package, version 8.0 (StatSoft 2007).

Results and Discussion

In table 2 are shown the parameters of the adjusted models and the statigraphs showing the goodness of fit of each model. In all cases significant values in the adjustments were observed, with coefficients of determination higher than 98 %. The typical error of the estimations was between 1371 and 1711 g. Statistically, from these results, the proposed models can be considered as appropriate. However, it must be highlighted that for the case under study, the logistical model and the growth curves of the females, in a general way, showed better adjustment with lower typical errors.

Abreu *et al*. (2004) refer that there is not defined method of comparison between models, which leads to the evaluation of parameters indicators of the regression. However, Rebato and Rosique (1994) suggest applying the sum of residual squares, when samples of the same size are available and using the typical error of the estimation when it is about populations of different sizes.

Considering the statistical analysis of these models, there is the possibility of estimating the performance of diverse variables of zootechnical interest during the

Table 1. Estimations of zootechnical interest

Estimated variable	Mathematical interpretation
Adult live weight when time tend to infinite (kg)	Parameter *a* of both models. Horizontal Horizontal asymptote $\lim_{x\to\infty} f(x)$.
Potential adult live weight when time tends to infinite (kg)	Upper limit of the confidence interval (95 %) for parameter *a*.
Live weight at weaning (kg)[1].	Evaluation of the model in x = 120 $f_{(120)}$
Age of highest growth rate (d)	Inflection point. Value of x that makes 0 the second derivative of the functions $f''(X_0) = 0$
Highest growth rate (g/animal/d)	Evaluation of the first derivative for *x* in the inflectionm point $f'(X_0)$
Weight at the age of highest growth rate (kg)	Evaluation of the model for x_0 that makes 0 the second derivative $f(X_0)$

[1]It is assumed weaning at 120 d from the management criteria for this genotype

Table 2. Adjustment indicators of the Gompertz and logistical models for growth of Creole kids

Indicator		Models			
		Gompertz		Logistic	
		Males	Females	Males	Females
Parameters (± typical)	a	40373.00	± 37.00	34297.00	± 564.00
	b	1.32	± 0.036	1.28	± 0.035
	c	8.410^{-4}	$\pm 310^{-4}$	8.710^{-4}	$\pm 310^{-4}$
Significance of the regression		***	***	***	***
Coefficient of determination (R^2 x 100 %)		98.07 %	98.09 %	98.29 %	98.32 %
Sum of residual squares		518416464.00	377818782.00	459804545.00	332460593.00
Typical error of the estimation (g)		1711.00	1461.00	1612.00	1371.00

Data for the analysis are expressed in g.

growth periods of kids. The adult weight, when time tends to infinite, is in correspondence with parameter (a) of the models. In this case, it was applied for estimating the adult adjusted live weight value of the Criolla goat by sexes, when time tends to infinite (Casas *et al*. 2009). From this analysis, males can grow, according to the logistical model, as far as 36.522 and females up to 31.354 kg. According to the Gompertz model, growth can be up to 40.373 and 34.297 kg, in males and females, respectively. The logistical model approaches more to the adult live weight means observed by La O *et al*. (2010) in this same population of Criollas goats (36.533 kg in males and 31.716 kg in females).

The upper limit, with 95 % confidence of this same parameter *a* (table 3), can be considered as a potential adult live weight indicator of the genotype under the management conditions of this population. According to the logistical model, maximum growth when time tends to infinite could be increased to 37.219 kg in males and to 31.964 kg in females, if the rest of the variables are favorably combined. If growth is understand in a simple way, as a net and progressive accumulation process of nutrients and their metabolites in the organism, then the maximum adult live weight when time tends to infinite has a genetically defined limit, and their variations are linked, mainly, to changes in the cell size (González and Pereda 2004).

According to the logistical model, the age at which the highest growth rate is verified (inflection point), is of 189 and 181 d (males and females, respectively), with gains of 135 and 118 g/animal/d, and live weights of 18.26 kg for males and 15.67 kg for females. The Gompertz model evidence a trend to move forward this moment, approximately to 147 d, with live weights and mean gains lower to those estimated by the logistical model.

The Cuban Criolla goat is included in a common pattern for the Western Caribbean (La Española, Jamaica and Cuba) and has much in common with the Criollas goats described in México, Central America and Venezuela. These goats are generally of small size type, compact body and strong extremities, characteristics resulting from their native stocks at the Iberian Peninsula, the African continent and the Canary Islands (Chacón 2009).

The live weight adjusted in this study, according to the models used, is lower to that described for Iberian goats by Gómez *et al*. (2009), who reported live weights between 55 and 80 kg for males and between 40 and 80 kg for females, in goats Celtiberian white, natives of Murcia-Granada and dark brown. Nonetheless, it is within the ranges described by Vargas (2003) for Criollas goats of Puebla in Mexico (11-47 kg). Revidatti *et al*. (2011) indicated weaning weights of 8.7 kg in Argentine Criollo kids. In Venezuela, the different Canary goat crosses with Criollas goats show a growth expression slightly higher at weaning, with 11.8 kg of live weight and adult live weight for the goats below 30 kg (Salvador *et al*. 2009).

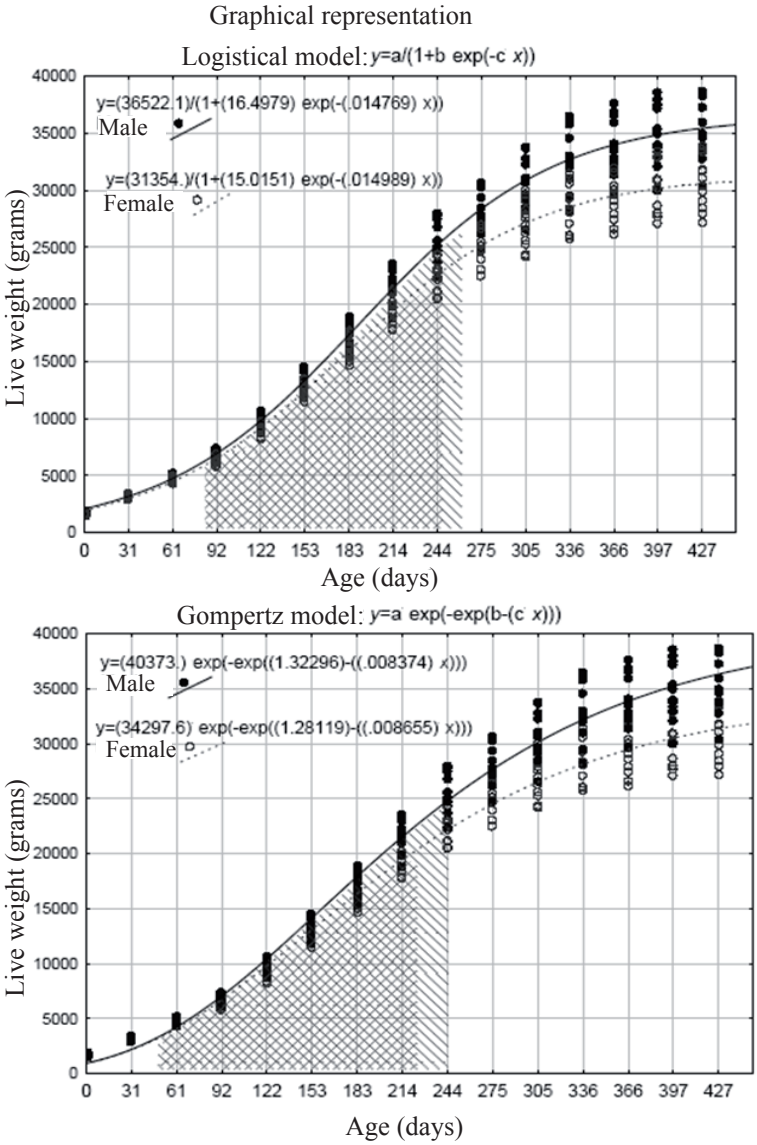
On evaluating the models during the growth stage of Criollas goats (figure 1), the adjusted logistical model started to be elastic from 82 d of age for both sexes, with mean daily gains higher than 70 g/a/d. It was extended to 260 d in the males and 247 d in females. In this latter stage, males and females attain live weight gains of 104 and 93 g daily, respectively. In females the growth peak remains 17 d less, which together with its lower live weight gain determines the growth difference. The adjusted Gompertz model shows tendency to advance and reduce this elasticity period, although gains at the beginning and end are similar to the logistical model. From the viewpoint of the zootechnical management, this period in which the animals express their maximum biological potential of growth is very important since it

Table 3. Estimated performance of some variables of zootechnical interest during the growth of Creole kids from the adjusted models

Variables[1]	Estimated performance			
	Gompertz		Logistic	
	Males	Females	Males	Females
Adult live weight when time tends to infinite (kg)	40.373	34.297	36.522	31.354
Potential adult live weight when time tends to infinite (kg)	41.829	35.412	37.291	31.964
Live weight at weaning[2] (kg)	10.213	9.587	9.601	8.996
Age of higher growth rate (d)	147.0	147.0	189.0	181.0
Highest growth rate (g/animal/d)	119.0	110.0	135.0	118.0
Live weight at the age of highest growth rate (kg)	14.852	12.617	18.261	15.677

[1]To facilitate the biological interpretation, the coefficient *a* and the model have been divided by 1000, transforming the original scale from g to kg

[2]Age at 120 d



Elasticity period

	Initial	Final
Age (d)		
	82m	260m
	82f	247f
Estimated live weight[1] (kg)		
	6.175m	26.961m
	5.814f	22.880f
Estimated mean daily gain (g/animal/d)		
	76m	104m
	71f	93f

	Initial	Final
Age (d)		
	48m	241m
	49f	225f
Estimated live weight[1] (kg)		
	3.275m	24.513m
	3.250f	20.519f
Estimated mean daily gain (g/animal/d)		
	69m	102m
	66f	91f

[1]To facilitate the biological interpretation, the coefficient a and the model have been divided by 1000 transforming the original scale from g to kg
m h Indicate sex of the animals (males, m and females, f)

Figure 1. Elasticity periods of the adjusted models and growth performance of the Criollas goats

indicates the moment when the rearing system must be perfected to reach better productive results.

In rearing systems, the definition of the period of highest growth expression supports management decisions for the herd. For the Criolla goat, this period varies between 144 and 275 d of age, which allow suggesting this moment as the adequate for finalizing kid fattening. From the biological point of view, a favorable combination of hyperplasia processes and cellular hypertrophy is corroborated (González and Pereda 2004). This is potentially defined by genetic factors, but is expressed by their best combination with environmental variables. La O *et al*. (2010) consider that for the Cuban Criolla goat this growth period can be restricted by the traditional rearing system, with basic objectives of self-survival and strategic marketing.

It is concluded that for the elasticity analysis, the application of the logistical and Gompertz models is effective for estimating the limits and duration of the growth peak period in Creole kids. This period in Cuban Creole kids generally extends from 165 to 193 d and ends between 225 and 260 d of age, stage when the animals reach an approximate live weight of 20 kg for females and 26 kg for the males.

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